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### Abstract

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# 1 Objectives

Our research program aimed at developing and applying theoretical, numerical, and graphical tools for the quantitative description and deeper understanding of stability and transition in technologically important flows at high supersonic speeds. Much of the effort has been directed toward engineering methods for advanced design and therefore spans a wide spectrum of concerns from basic transition issues to the efficiency and usability of software in the engineering environment. Our work has concentrated on the following areas:

- (1) Improvements of formulation and numerical solution of the parabolized stability equations (PSE) in general curvilinear coordinates.
- (2) Efficient numerical methods to solve basic-flow, stability, and transition problems
- (3) Transition analysis in high-speed flows
- (4) Disturbance environment and receptivity studies.
- (5) Stability theory for 3D boundary layers.
- (6) Interactive simulation and visualization of flows in distributed computer environments.

# 2 Achievements

The work in areas (1) and (2) has been completed. We currently have two versatile PSE codes available. The first code is very flexible to adapt to 2D or 3D convectively unstable flows. The code is restricted to Cartesian coordinates and primarily serves for producing benchmark results and exploring new concepts. The second code is developed in cooperation with DynaFlow, Inc. and is written specifically for supersonic boundary layers in realistic geometry. This code is the basis for specific applications e.g. to high-Mach-number flows over blunt cones. The work with this code is supported by various other codes to compute the inviscid and viscous supersonic boundary-layer flow. The codes have been utilized to perform successful studies in areas (3) and (4). Whenever possible, results were compared with experiments and other (DNS) computations. Special attention has been paid to requirements for basic-flow solvers for practical applications.

In area (5), an attempt has been made to extend the stability theory and transition analysis to fully 3D boundary layers, which previously have been treated only under the parallel-flow approximation, while the PSE account

for streamwise but not for spanwise variations. This work has led to the conclusion that extended PSE simulations may be as costly as DNS of the nonlinear stability equations. Therefore, we have analyzed in area (6) the concept and feasibility of an advanced code to perform simultaneous simulation and visualization of unstable and transitional flows on distributed computer systems. Code design and implementation were not part of this project.

Two graduate research assistants and one postdoctoral associate cooperated in the work under this contract. The difficulty of the research areas listed above has required a major involvement of the principal investigator. Details on progress in the various areas of interest are reported below.

## 2.1 PSE in General Curvilinear Coordinates

Previous work has either neglected or only partially accounted for the curvature effects on basic flow and stability characteristics. While the step-by-step inclusion of curvature in previous work has provided the opportunity for follow-up publications of minor value, we have decided to account for the curvature terms completely and from the beginning. We have derived the nonlinear stability equations including transverse and longitudinal curvature as functions of the distance from the wall. For the highly compressible flows of interest, the approximation to the thermal properties and nonlinear terms have been improved by a consistent formulation up to fourth order. The linearized equations have been coded for use with both spectral method or finite-difference methods. The coded insert files are similar to the file format used by the stability code `linear.x` [1] and consistent with the more advanced and efficient stability code LISA developed at DynaFlow. These files can be directly incorporated into the PSE code.

The testing of these codes has been a problem in itself since there are rarely any reliable data to compare with. To guarantee correct function, we have devised a suite of tests that solves the same group of problems in different coordinate systems. The results of these tests are not expected to completely agree but differences must be within the known approximations involved in the analysis.

## 2.2 Numerical Aspects

For flows in realistic geometries, various procedures have been coded and compared to obtain the metric terms and flow quantities as well as their derivatives as accurately as possible. This task is non-trivial since the finite-difference methods used for calculating the basic flow are usually only of second order in space. Grid points chosen for the basic-flow calculation are

not necessarily suited for stability analysis. Therefore, special interpolation routines have been written to account for the truncation errors introduced by finite-difference methods and the usual fixed-format output of data.

In earlier work for low Mach numbers we have successfully used spectral methods to solve the original equations directly. When using similarity variables, this approach still works at high Mach numbers. In the physical coordinates required by the PSE technique, however, spectral methods are not suited because of the inherent clustering of collocation points near the wall while the dominant second modes require high resolution near the edge of the boundary layer.

An alternative are compact finite-difference methods which have been successfully exploited for the linear stability analysis of similarity solutions to the boundary-layer equations. In realistic flows, these methods are less useful since they require higher derivatives of the basic-flow quantities. These derivatives are inaccurate and difficult to obtain from computational solutions. The second severe disadvantage of the compact method is the need to write the equations as a first-order system. For the PSE, the first-order system introduces the inverse of the transverse component of the basic flow into the coefficients of the differential equations. This transverse component is generally small and difficult to obtain by finite-difference methods. Moreover, the transverse component can change sign and cause undesirable singularities of the problem. We have therefore performed extensive studies of high-order numerical methods suitable for boundary-layer, stability, and PSE calculations.

### 2.2.1 Hermitian Methods

The goal of our efforts was to find finite difference schemes that are of high order, can be applied to second- and third-order differential equations, lead to block-tridiagonal systems (for efficient solution), and work for nonuniform grids. The result of this work was the selection of Hermitian methods which involve only two or three neighboring points. The use of the function values and derivatives at these points introduces enough degrees of freedom to reduce the truncation error to any desirable order.

Hirsh [2] has used Hermitian high-order finite-difference schemes to solve some fluid mechanics problems. He attributes the scheme to "a suggestion made by Kreiss, pertaining to a new compact differencing of fourth-order accuracy." The idea of formulating high-order finite-difference approximations in a compact way, i.e. by utilizing the values of derivatives instead of an increasing number of neighboring function values, dates in fact back to the work of Hermite [3]. Selected formulas are tabulated by Collatz [4, Table III, pp. 538ff] who also discusses Hermite's method. Both Collatz and Hirsh give expressions only for equidistant grid points. To maintain high order,

special attention has to be paid to the boundary conditions. To achieve the same 4th-order approximation for interior and boundary points, Hirsh uses boundary formulas that involve more than the three neighboring points in the interior, although this procedure prevents the use of the highly efficient solvers for block-tridiagonal systems.

The use of Hermitian methods on coarse and nonuniform grids is discussed by Adam [5]. Adam uses boundary formulas of lower order than in the interior and uses nonuniform grids to improve the accuracy near the boundaries. He concludes that the lower-order boundary formulas have no detrimental effect on the accuracy of the solution. However, Adam's utilization of nonuniform grids is in conflict with our desire to use the grid spacing to account, and increase the resolution, for physical phenomena such as internal boundary layers.

We have developed a systematic procedure to generate all linearly independent Hermitian formulas using up to the third derivative. For a system of  $N$  second-order differential equations, two of these formulas are needed at every interior point, three at the boundary. By exploiting the null space of the governing system of Taylor expansions, we have constructed methods of at least sixth order which increases to seventh order as the number of grid points increases. By an elimination procedure, the original dimension  $3N$  of the blocks in the diagonal system can be reduced to  $2N$ , the same number as with the compact method. Unavoidable deviations from tri-diagonality at the boundaries require a dedicated subroutine for solving the algebraic system. In contrast to the usual solvers for banded matrices, our subroutine exploits the block structure and increases the accuracy of the solution by partial pivoting within blocks.

The Hermitian method has been completely developed for systems of second and third order. Special elimination procedures have been written for systems derived from the Navier-Stokes equations which have no boundary conditions on the pressure. The method matches the accuracy of a spectral method at a fraction, say 2-5% of the computer time and reduced demand for memory. The method is key to using the PSE code on workstations, a definite advantage for practical applications. A report on this method has been prepared and will be refined for publication as time permits.

### 2.2.2 Improved PSE Code

A detailed performance analysis of typical PSE runs has been conducted to identify areas with high payoff of improvements. These improvements can be achieved by changing formulation, numerical algorithms, and implementation. Formal changes have concentrated on the norm necessary to decompose disturbance modes into a slowly varying profile and a wave function with slowly varying wavenumber. The choice of the norm affects the convergence

of this iterative process which is at the basis of the PSE approach. Since no rigorous mathematical means are available for this nonlinear problem, intuition and trial-and-error have been applied and led to considerable improvements. The formulation of the far-field boundary conditions has been modified to better describe the asymptotic behavior of the solution and hence allow for a smaller computational domain. Major changes have been made in the formulation and implementation of the algorithms to iteratively solve the nonlinear PSE. These changes improve the convergence and allow to proceed further into the breakdown process.

Instead of using a nonuniform grid to solve the differential equations in the original variable normal to the boundary, we have implemented the stretching by a transformation of the wall-normal variable. The transformation affects only the coefficients of the Hermitian formulas. Using uniform grids to solve the differential equations in the transformed variable is more efficient and more accurate.

The previous numerical methods to solve the algebraic problems obtained by spectral or Hermitian finite-difference method have been replaced by LAPACK routines that make extensive use of the BLAS routines. Hand-coded optimized BLAS routines are available on supercomputers and some workstations (e.g. SGI). The number of costly evaluations of the nonlinear terms and the effort per evaluation have been reduced though on the expense of additional memory. Additional savings have been achieved by structuring the data for specific computer architectures. The use of cache on workstations requires array indices arranged opposite to those for vector processors.

The current PSE codes are faster, more robust, and have a wider range of application than the previous generation.

### 2.3 Stability and Transition Studies

To guarantee proper functionality of the linear stability code LISA and the PSE code, we have repeated various published calculations in the speed range from  $Ma \approx 0$  to  $Ma = 10$ .

Figure 1 shows the result of a convergence study for test case IV of Malik [6] at  $Ma = 10$ ,  $Re = 1000$ ,  $T_e = 111.111^\circ K$ , and  $\alpha = 0.12$  for the compressible flat-plate boundary layer. Our "exact" result for the temporal eigenvalue was obtained by the spectral method with  $J \geq 50$  as  $\omega = 0.115861436 + i0.000137657$  (different from the spectral results given in ref. [6]). The error comprises both the errors of the basic-flow calculation and stability analysis. To obtain the correct eigenvalue within an absolute error of  $10^{-6}$  or  $10^{-8}$  requires about 100 or 200 grid points, respectively. The spectral result is matched with 300 points. At lower Mach numbers,  $Ma \leq 2$ , good accuracy can be achieved with as few as 50 grid points. At lower Mach



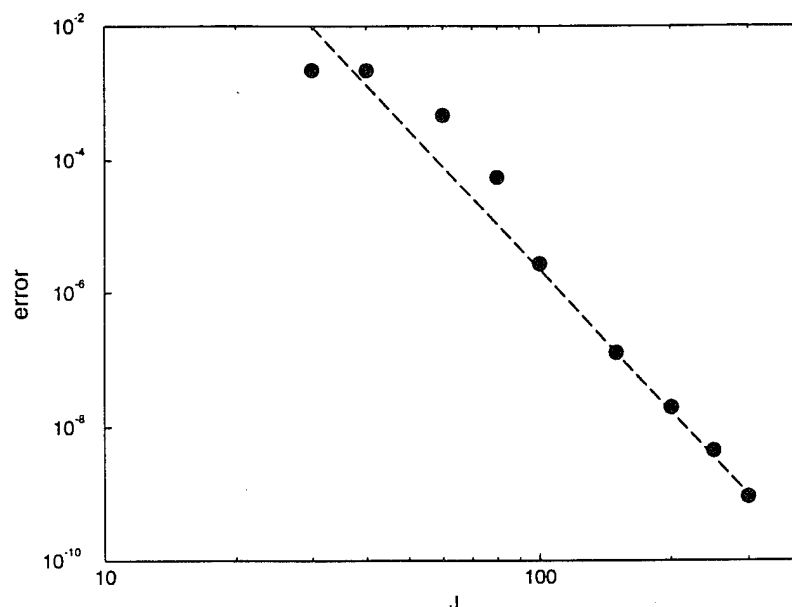


Figure 1: Error of the temporal eigenvalue calculated with the three-point Hermitian method at high Mach number. The dashed line indicates seventh-order accuracy.

numbers, it is sufficient to use two-point Hermitian methods which allow 4th-order accuracy. The smaller bandwidth of the resulting algebraic systems increases the efficiency of the computations.

Figure 2 shows the comparison with results of El-Hady [7] for subharmonic secondary instability in the flat-plate boundary layer at  $Ma = 1.2$ . Two PSE runs were performed using Cartesian and similarity coordinates, respectively. Although the PSE runs have been tailored to match the locally-parallel flow assumption, the results deviate from El-Hady's due to a slight shift of the instability region for the primary mode toward higher Reynolds number  $R$ . Except for the initial transients, the PSE results for the solutions in different coordinates agree. The transients are stronger in Cartesian coordinates because the initial data are more affected by the parallel-flow assumption of the LST.

Various other published results on the temporal evolution of secondary instabilities and first-mode/second-mode interactions at higher Mach numbers have been repeated with the PSE code which provides the spatial evolution. The differences of spatial and temporal evolution are remarkable. The temporal strong interaction, e.g. with the subharmonic, is sustained by holding the parameters fixed. In the spatial evolution, resonances occur locally and are quickly detuned as the disturbance waves travel downstream into a range of different Reynolds number and boundary-layer flow. Consequently, the inter-

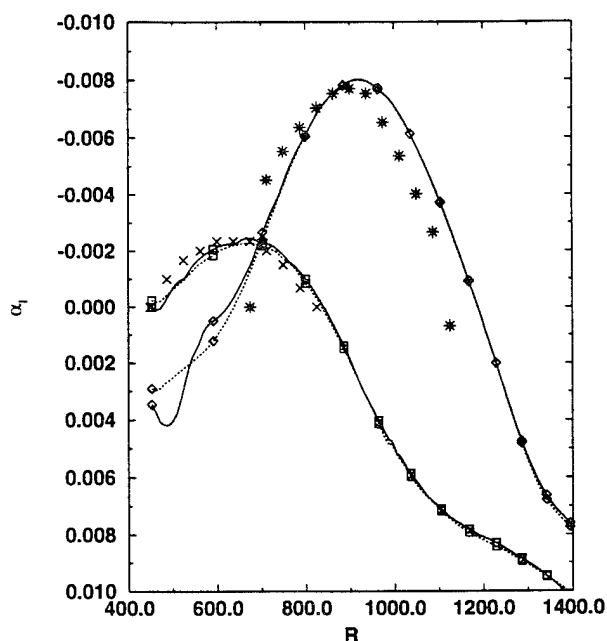


Figure 2: Subharmonic secondary instability at  $Ma = 1.2$ . Comparison of the PSE solution in Cartesian (solid) and similarity coordinates (dashed) with results of El-Hady (symbols).  $\square$ : PSE, primary mode (2,0),  $\diamond$ : PSE, subharmonic mode (1,1).  $\times$ : El-Hady, primary mode (2,0),  $*$ : El-Hady, subharmonic mode (1,1).

actions are much weaker than in the temporal simulations, and the transition mechanisms that dominate low-speed transition may not be responsible for transition at high speeds.

### 2.3.1 Flow over Swept Wings

Flows over the swept wings of airplanes are routinely analyzed for the transition location using the  $e^N$  criterion [8, 9]. The  $N$  factor is obtained by one of a series of proprietary or restricted stability codes. The  $e^N$  code COSAL by Malik [10] is in widespread use in the U.S.<sup>1</sup>

We have performed a detailed comparison of amplitude-growth curves ( $N$  factor versus arclength) obtained with different options of COSAL, our own stability code LISA, and the linear PSE code [11]<sup>2</sup>. An infinite wing with a 64A010 airfoil and sweep angles of  $0^\circ$ ,  $25^\circ$ ,  $53^\circ$ , and  $70^\circ$  was considered at  $Ma = 1.5$  and an altitude of 55,000 feet.<sup>3</sup> The modified version WING

<sup>1</sup>COSAL was provided by NASA Langley Research Center for our study.

<sup>2</sup>The computations for realistic geometries were performed in cooperation with G. Stuckert and N. Lin, DynaFlow, Inc.

<sup>3</sup>The Euler solutions for the various cases were provided by G. Klopfer, NASA Ames

of the boundary-layer code by Kaups & Cebeci [12] which is provided with COSAL was used to compute the basic flow for all stability computations. The flow over this slender airfoil exhibits weak TS instability at  $0^\circ$  sweep and is otherwise dominated by cross-flow instability.

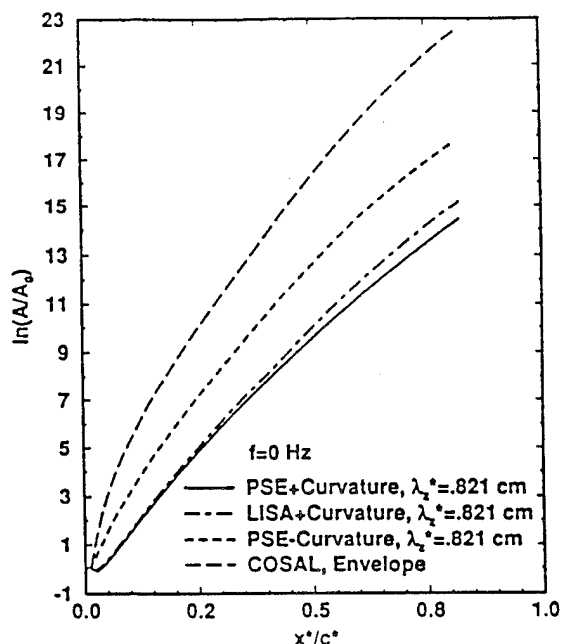


Figure 3: Amplitude growth curves for cross-flow vortices obtained with COSAL (envelope method), PSE without curvature effect, LISA, and PSE.

Figure 3 compares the amplitude growth of steady cross-flow vortices at  $53^\circ$  sweep obtained with the envelope option of COSAL, the PSE without accounting for curvature effects, LISA, and the PSE. COSAL does not account for curvature. The largest deviation between the results is caused by the violation of physical constraints by the envelope method which at any point selects the spanwise wavenumber for maximum growth while physical solutions have fixed spanwise wavenumber [13]. Although our version of COSAL provides alternative strategies, the physical constraints cannot be satisfied. The second major change is the strong stabilizing effect of surface curvature. (The PSE approach does not suffer from the inconsistency between in-plane curvature and parallel-flow approximation [14, 15].) The discrepancy between results of LISA and PSE code reflects the small effect of nonparallelism. While the discrepancy is small in this case, it is significant for other airfoils, and the growth predicted by LISA may be substantially higher or lower than that obtained with the PSE code.

The comparison of the different results for  $N$  factors enhances the critical attitude toward  $e^N$  transition predictions and raises doubts in the value of the extensive database of  $N$  factors created by use of COSAL and other stability codes.

Other studies on swept wings [16] aimed at comparing receptivity and disturbance evolution with the low-speed experiments of Radetzsky et al. [17, 18] on a  $45^\circ$  swept wing with an NLF(2)-0415 airfoil.<sup>4</sup> The flow over this airfoil is one of the cases where LISA predicts a substantially larger amplitude growth than the PSE code.

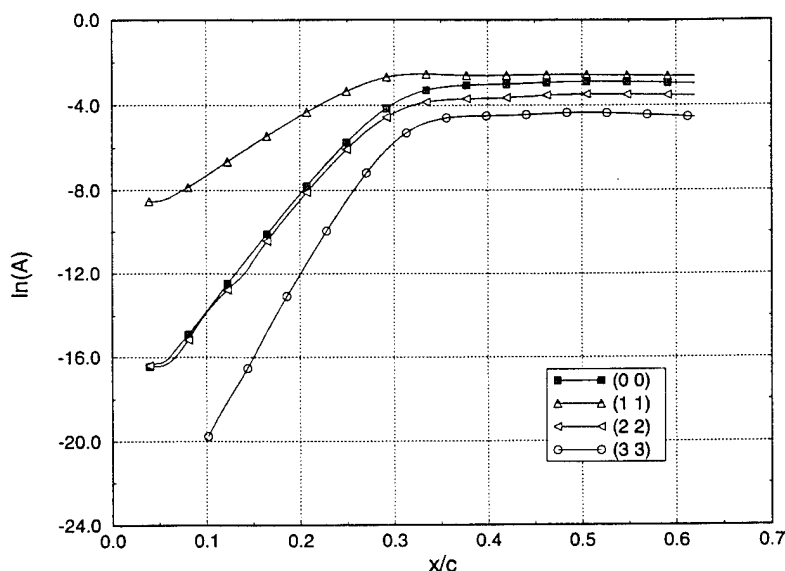


Figure 4: Nonlinear growth of an unsteady cross-flow vortex with  $f = 180$  Hz,  $\beta = 0.4$ , and an initial amplitude of  $A_0 = 0.0002$  at the neutral point in the flow over the NLF(2)-0415 airfoil.

Figure 4 shows the amplitude growth of an unsteady cross-flow vortex calculated using 3 harmonics. Owing to the large growth rate, the vortex reaches early saturation near 30% of the chord. Steady vortices of the same initial amplitude saturate downstream of 60% chord at a higher amplitude level. Nevertheless, unsteady vortices create a stronger mean-flow distortion. As in swept Hiemenz flow, the saturation amplitude is largely independent of the initial amplitude, however, it varies with the spanwise wavenumber  $\beta$ . A more detailed comparison with the experimental data of Radetzsky et al. based on the measured pressure distribution was planned. Up to this time, however, we were unable to obtain the measured pressure distribution from

<sup>4</sup>The geometry and computational results for the pressure distribution at  $-4^\circ$  angle of attack were provided by W. S. Saric.

W. S. Saric.

### 2.3.2 Flow over a Blunt Cone at $Ma = 8$

The stability of the flow over a blunt cone at  $Ma = 8$  has been experimentally studied by Stetson et al. [19] for different nose bluntness. The cone has a half angle of  $7^\circ$ , a length of 40 inches, and a base diameter of 9.823 inches. The stability of the mean flow over the blunt cone with a 0.15-inch nose tip is considered here, because this case is controversial and has developed into a benchmark case for application of basic-flow and stability codes. While the computational tools provide converging results, the discrepancy between stability results for sharp and blunt cones in the same experimental facility and the disagreement of the blunt cone results with stability calculations remain unexplained.

The basic flow has been calculated using two different codes: a Thin Layer Navier-Stokes (TLNS) code developed by Esfahanian [20, 21, 22] and the AFWAL Parabolized Navier-Stokes (PNS) code. The TLNS solution has been computed to serve as an accurate benchmark.

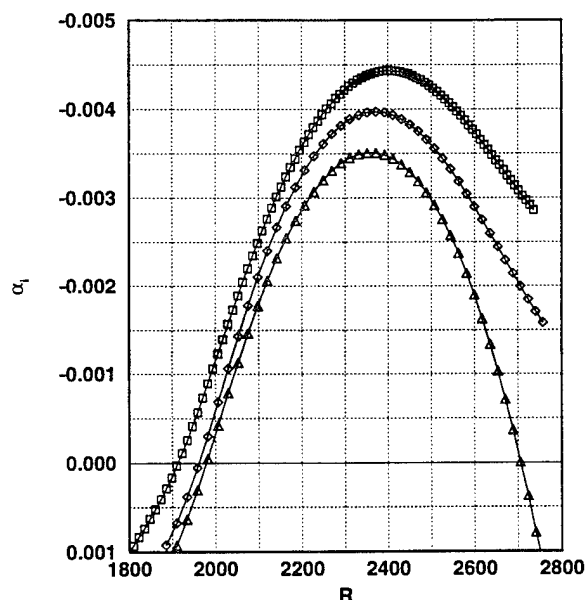


Figure 5: Comparison of spatial growth rates as a function of arc length using different solutions for the basic flow.  $F = 82.7 \cdot 10^{-6}$ .  $\square$ : TLNS solution, 200 wall normal points.  $\diamond$ : AFWAL PNS solution, 501 radial points.  $\triangle$ : AFWAL PNS solution, 251 radial points.

Figure 5 compares the spatial growth rates for the two-dimensional second mode at  $F = 82.7 \cdot 10^{-6}$  obtained with different basic flows. Although the PNS code has been pushed to the limits with 501 radial points (and an

automatic increase in the number of streamwise stations), there remains a significant difference from the growth rates obtained with the TLNS solution. This difference is enhanced by integrating the growth rates into  $N$  factors. Comparison of basic-flow quantities shows in general good agreement but differences near the edge of the boundary layer. Since the critical layer of the second mode is located in this region, the stability results are strongly affected by the accuracy of the solution.

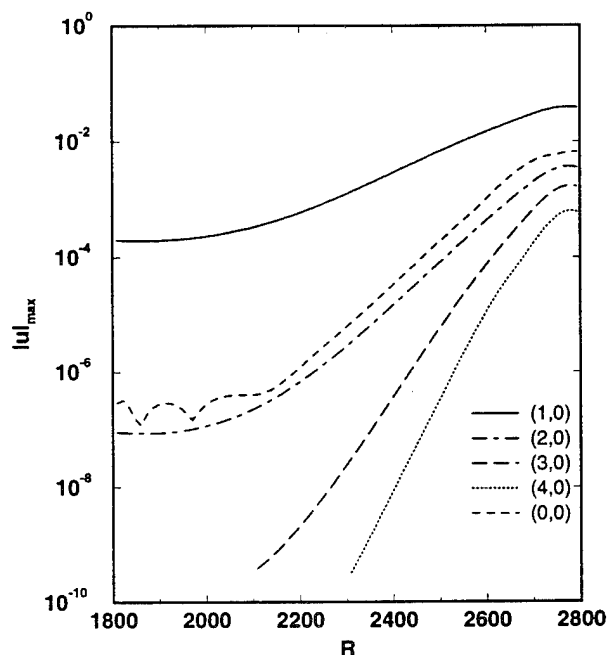


Figure 6: Nonlinear evolution of a 2D second Mode at  $F = 82.7 \cdot 10^{-6}$  for an initial amplitude of 0.0002.

The nonlinear evolution with 4 harmonics of a two-dimensional second mode at  $F = 82.7 \cdot 10^{-6}$  and a small initial amplitude of only  $2 \cdot 10^{-4}$  is shown in figure 6. The TLNS solution has been used as basic flow. In the linear case (figure 5), the second mode exhibits growth well beyond  $R = 2800$ . Here the nonlinear terms cause a stabilization starting near  $R \approx 2500$  and decay of the mode for  $R > 2760$ . In spite of the moderate maximum amplitude, the disturbance causes a significant rise in the skin friction coefficient  $C_f$  that dramatically increases for larger amplitudes.

We have studied various second-mode and first-mode interactions to identify resonance mechanisms and drive the flow into transition. Temporal analysis of subharmonic transition at  $Ma = 4.5$  [23, 24, 25] has shown a strong resonance between a two-dimensional second mode and a three-dimensional subharmonic first mode. In the blunt-cone flow, spatial analysis of this mechanism has only a weak effect. The growth of the subharmonic mode is en-

hanced only in a small range of Reynolds numbers, and as the second mode grows, it suppresses the growth of the first mode below the linear level until it decays. To clarify this unexpected observation, we performed various spatial simulations of the subharmonic “transition” at  $Ma = 4.5$  in the flat-plate boundary layer. The parameters were chosen to match those of the temporal studies and later varied in their neighborhood. We found that the lack of a strong subharmonic resonance is not particular to the blunt-cone flow. In the spatial case, enhanced growth of the subharmonic is limited to a short region, too short to increase the amplitude of the (assumed) small subharmonic mode to a level where it would strongly interact with the two-dimensional mode. As in the temporal case, we found no evidence for fundamental secondary instability.

Of all the cases studied in the flow over the blunt cone, the strongest interaction and growth was observed for an interacting pair of oblique second modes with  $F = 82.7 \cdot 10^{-6}$  and an initial wave angle of  $40^\circ$ . With initial amplitudes of  $2 \cdot 10^{-4}$  at  $R = 2000$ , breakdown occurs near the end of the cone. The longitudinal vortex components exhibit strong growth. To pursue this case further, we will need to extend the basic flow computation further downstream from the end of Stetson’s cone.

### 2.3.3 Flow over a Sharp Cone at Angle of Attack

Linear stability calculations have also been performed for the windward and leeward meridian of a sharp cone at  $2^\circ$  angle of attack. While the tendencies are consistent with the experimental data, we found significant differences between results of LISA and PSE code. For reasons of symmetry it is clear that the waves propagate along the meridians. There are, however, significant variations of the basic flow in both the marching and azimuthal direction. Although we derived some evidence that the PSE approximation and PSE results are valid, we concluded that a more detailed analysis of the stability of fully 3D boundary layers is necessary before progressing into the flow away from the symmetry plane. A report and discussion of these results has been given by Stuckert et al. [26].

## 2.4 Disturbance Environment and Receptivity

The difficulty of identifying strong transition mechanisms in the  $Ma = 8$  flow over a blunt cone suggests that there may be strong receptivities to certain disturbances of the free-stream or the surface that cause elevated disturbance levels. Theoretical work and understanding in this area are rare, especially at high but finite Mach numbers. Since the PSE permit internal forcing or forcing at the boundaries, we have started to study receptivity with a

properly modified PSE code. This work<sup>5</sup> has led to significant results for incompressible flows for which experimental data are available.

### 2.4.1 Klebanoff Modes

The first area of receptivity we pursued is the generation of steady or low-frequency longitudinal vortices observed in numerous experiments since Dryden in the 1930th. Following the detailed description of this phenomenon by Klebanoff [27] these disturbances are called "Klebanoff modes". Recent work by various groups has shown that the non-selfadjoint systems of stability equations for shear flows support the transient temporal growth of certain disturbances not associated with unstable eigenvalues. The transient growth that precedes the ultimate decay may be by orders of magnitude and is typically strongest for longitudinal vortex modes in parallel flows. The analysis requires solving an initial-boundary value problem in time. All previous studies have specified initial conditions while the boundary conditions were homogeneous.

In the light of receptivity, a more realistic approach is to solve an initial-boundary value problem in streamwise direction with inhomogeneous boundary conditions: a good problem for a slightly modified PSE code. Our results [28] have not only explained the origin of Klebanoff modes and their parametric characteristics but also shown a direct link to Görtler vortices and cross-flow vortices which all share similar receptivity mechanisms [29]. Instead of the ultimate decay at higher Reynolds numbers, the Klebanoff modes are likely to couple with unstable waves and lead to breakdown.

The strong growth of longitudinal vortices in the oblique-mode interaction on the blunt cone in presence of small nonlinear forcing indicates that there may exist "supersonic Klebanoff modes" which are difficult to detect with the experimental techniques applied so far. The results of an analysis of this new mechanism in the compressible flat-plate boundary layer are forthcoming and the study will be continued for the blunt-cone flow. There are good reasons to believe that the transient growth mechanism is active at all Mach numbers. Without deeper understanding of the area-distributed receptivity for Klebanoff modes, and their connection to the vortical disturbances of free-stream turbulence, we have not yet pursued this mechanism at supersonic speeds.

The analysis of the receptivity to vortical disturbances is near completion for 2D disturbances. In this case, wall curvature, roughness, or waviness is required to provide a streamwise length scale. The 3D case is still under study.

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<sup>5</sup>partially funded under other contracts



### 2.4.2 Model Environments

Besides the receptivity mechanisms, quantitative account for the internalized disturbances requires knowledge of the disturbance environment. Our efforts have focused on characterizing the proper disturbance environment in atmospheric flight or different wind tunnels. Very limited measurements are available for wind tunnels (typically the turbulence level) while the operating environment of advanced flight vehicles is unknown. The work in this area essentially involves the construction of model-data sets as input to the codes based on mechanisms and scales of instabilities and comparison of the results with observed transition points. Uncertainty arises both from insufficient knowledge of the environmental disturbances at the boundary (vibrations, roughness, waviness) or in the free stream and the receptivity of the flow which determines the associated disturbance amplitudes inside the boundary layer.

We have studied different types of model data. In various test runs, we have replaced the generation of initial data by inhomogeneous conditions at the initial position. This approach is similar e.g. to the introduction of wave packets in experiments or DNS runs. A pulse, for example, introduces a spectrum of disturbance waves, all in proper phase relation, and allows detailed studies of the transient evolution and ultimate growth or decay. A PSE run to "repeat" the experiment of Gaster and Grant [30] is completely set up. The run has not yet been executed since the post-processing of the data will be time-consuming and sufficient disk space to hold the data on-line was unavailable until recently. Studies of harmonic point sources [31, 32, 33] have been performed in cooperation with L. Mack, JPL, and published by Mack & Herbert [34]. These receptivity studies are in direct context with flow control and the analysis of actuator concepts. The insight into the transient flow originating from an actuator is important for the design of experiments to explore transition in supersonic flows.

## 2.5 Stability of 3D Boundary Layers

Detailed studies have been conducted to find the correct solution to the problem of normal-mode evolution in 3D boundary layers. Controversial views on this subject have been presented by Cebeci, Mack, Malik, and Nayfeh in their papers and at a workshop of the US Transition Study Group. At this time, a clear answer has been found only for boundary layers with 3D velocity over 2D geometry, e.g. infinite or conical wings. Our findings are in essential agreement with Mack. For an infinite wing, the spanwise wavelength of disturbances must remain constant. This constraint is violated by most stability codes for practical applications, in particular by all optional strategies of our version of COSAL. Comparison of stability results for the

same boundary layer on an infinite and a conical wing shows significant effects of the three-dimensionality.

For the fully 3D problem, we have prepared a local stability analysis including basic-flow variations in both directions parallel to the surface. The formulation is ill-posed since one condition on the wavenumber variation is missing. The conservation principles used for tracing rays through non-dissipative systems are unusable in our situation. After intense study, we concluded that the problem originates from prescribing the initial condition only along a single line normal to the surface while the true solution requires specification in the 2D inflow area. It appears possible to trace this field with an adapted form of the PSE in the marching direction. The computational cost will significantly increase. Therefore, one may also consider DNS as a possible approach. Implementing any of our ideas to solve the stability problem for fully 3D flows will exceed the framework of this contract. Nevertheless, we have analyzed the requirements and feasibility of a new generation of CFD software.

## 2.6 Flow Simulation and Visualization

In our work on realistic configurations, we have observed a series of shortcomings of conventional codes for computations of steady, laminar basic flows, and their use in today's computing environment. The analysis of both Navier-Stokes solutions and Euler solutions for flows over wings and blunt cones has revealed unexpected flaws in these solutions. The present status of computers and the numerical methods used in common Navier-Stokes solvers do not allow sufficiently accurate solutions for the boundary layer. The high cost of using these codes is unaffordable in engineering practice. The need for supercomputers further limits the use of these codes for two reasons: the necessary resources and the unpredictable turnaround time in a shared computing environment.

The aerospace industry sees the combination of an Euler code with a boundary-layer code as the best way to obtain the basic flow. From the Euler solution, only the surface pressure is actually needed. We have used the special version of the Kaups-Cebeci code distributed by NASA for use with COSAL. Even in the solutions of this commercially used code, we found unacceptable inaccuracies. Earlier, we had designed a new code that solves an extended set of flow problems with Hermitian finite-difference methods. The implementation of the design was not intended within this project since we wanted to dedicate our resources to the fundamental aspects of transition prediction in practice. Meanwhile, we consider improvements in the computational tools as one of these fundamental aspects.

We estimate that most of the frequently used codes ignore some 20 years

of development in numerical methods and computers. With a concurrent research code [35], we have performed successful transition simulations in about one day on an SGI Indigo R4000 workstation. The same run was repeated on a Cray C90 in 3/2 hours run time and a few days wall clock time. The newest model of the Indigo runs at a peak floating-point rate comparable to a Cray YMP processor. Considered the 30+ Indigos in our department, we could easily beat the C90 today and analyze the flow while it is computed if the work could be efficiently distributed and provisions for the display of data would be available. With available virtual-reality equipment, we could actually "walk around" in the flow, and inspect the vorticity or the skin friction on the "floor". With a model problem, we have successfully tested the distribution of the work across four workstations using PVM (Parallel Virtual Machine). The PVM software is publicly available and already part of some operating systems, e.g. for the Cray T3D. Distributed interactive flow simulation and visualization could be a reality today for workstation clusters (available in industry) as well as massively parallel supercomputers such as the Intel Paragon.

We have started to draft the concept of such a code and to collect reusable components of various source software for grid generation, work distribution, computation, and visualization. In principle, the code can be designed to solve different sets of equations in different subdomains or runs. This would enable the use of the same code for computing the steady basic flow from the Navier-Stokes equations, the transition in this flow from the nonlinear stability equations, and the resulting temperature distribution within the body. We will pursue the development of this code alongside with the physical transition issues beyond this project.

### 3 Personnel

The following personnel has participated in the work and has been partially supported under this contract:

Th. Herbert, Principal Investigator

Rihua Li, Postdoctoral Associate

Mengjie Wang, PhD Student

Jinhui June Cui, MS Student

Charlotte Herbert, Systems Programmer 2

Dr. Li has resigned from his position on short notice without delivering the results of his work in documented form. June Cui has discontinued her graduate studies without advanced degree to care for her baby.

## 4 Degrees and Awards

Mengjie Wang received his Ph.D. for his Thesis

“Stability Analysis of Three-Dimensional Boundary Layers with Parabolized Stability Equations,” The Ohio State University, Columbus, Ohio, July 1994.

Thorwald Herbert has been invited to contribute an article on “Parabolized Stability Equations” to the next volume of *Annual Reviews in Fluid Mechanics*.

## 5 Publications

The following publications were completed, or originated from work under support by this contract:

“A Note on the Calculation of Landau Constants,” by J. D. Crouch and Th. Herbert, *Phys. Fluids A*, Vol. 5, pp. 283-285, (1993).

“Stability of Hypersonic Flow over a Blunt Body,” by Th. Herbert and V. Esfahanian, In: “Theoretical and Experimental Methods in Hypersonic Flows,” AGARD CP 514, pp. 28/1-12 (1993).

“Linear and Nonlinear Stability of the Blasius Boundary Layer,” by F. P. Bertolotti, Th. Herbert, and P. R. Spalart, *J. Fluid Mech.*, Vol. 242, pp. 441-474 (1992).

“Transition Studies on Realistic Configurations,” by Th. Herbert and G. K. Stuckert, *Bull. Am. Phys. Soc.*, Vol. 37, pp. 1753 (1992).

“Stability and Transition on Swept Wings,” by G. K. Stuckert, Th. Herbert, and V. Esfahanian, AIAA Paper No. 93-0078 (1993).

“Effects of Free-Stream Turbulence on Boundary-Layer Transition,” by Th. Herbert, G. K. Stuckert, and V. Esfahanian, AIAA Paper No. 93-0488 (1993).

“Nonlinear Evolution of Secondary Instabilities in Boundary-Layer Transition,” by J. D. Crouch and Th. Herbert, *Theoret. Comput. Fluid Dyn.*, Vol. 4, pp. 151-175, (1993).

“Studies on Boundary-Layer Receptivity with Parabolized Stability Equations,” by Th. Herbert and Nay Lin, AIAA Paper No. 93-3053 (1993).

“Nonlinear Analysis of Swept Wing Transitional Boundary Layers,” by G. K. Stuckert and Th. Herbert, SAE Paper No. 932515 (1993).

"Parabolized Stability Equations," by Th. Herbert, in: Special Course on *Progress in Transition Modelling*, AGARD Report No. 793, pp. 4/1-34 (1994).

"PSE Analysis of Receptivity and Stability in Swept Wing Flows," by M. Wang, Th. Herbert, and G. K. Stuckert, AIAA Paper No. 94-0180 (1994).

"Management of Crossflow Vortices on Swept Wings," by G. K. Stuckert, M. Wang, and Th. Herbert, AIAA Paper No. 94-1847 (1994).

"Crossflow-Induced Transition in Compressible Swept-Wing Flows," by M. Wang, Th. Herbert, and G. K. Stuckert, AIAA Paper No. 94-2374 (1994).

"Linear Wave Motion from Concentrated Harmonic Sources in Blasius Flow," by L. M. Mack and Th. Herbert, AIAA Paper No. 95-0774 (1995).

"Nonparallel Effects in Hypersonic Boundary Layer Stability," by G. K. Stuckert, N. Lin, and Th. Herbert, AIAA Paper No. 95-0776 (1995).

"Boundary Layer Receptivity to Freestream Vortical Disturbances," by N. Lin, G. K. Stuckert, and Th. Herbert, AIAA Paper No. 95-0772 (1995).

## 6 Technical Presentations

"Transition Prediction in High-Speed Boundary Layers," by Th. Herbert, Fluid Dynamics Laboratory, NASA Ames Research Center, Mountain View, California (January 1992).

"Stability of Hypersonic Flow over a Blunt Body," by Th. Herbert and V. Esfahanian, AGARD Symposium on *Theoretical and Experimental Methods in Hypersonic Flows*, Torino, Italy (May 1992).

"Transition Analysis with Parabolized Stability Equations," by Th. Herbert, DLR Göttingen, Germany (May 1992).

"Modern Developments in Numerical Simulation of Flow and Heat Transfer," keynote lecture, by Th. Herbert, 1992 National Heat Transfer Conference, San Diego, California (August 1992).

"Boundary-Layer Transition on Aerodynamic Configurations," by Th. Herbert, International Symposium on *Computational Fluid Dynamics for Air Vehicle Technology*, Wright-Patterson AFB, Ohio (September 1992).

"Studies on Swept Wing Transition," by Th. Herbert and G. K. Stuckert, SAE Aerotech '92, Anaheim, California (October 1992).

"Stability and Transition on Swept Wings," by G. K. Stuckert and Th.

Herbert, NASA Ames Research Center, Moffett Field, California (October 1992).

"Transition Studies on Realistic Configurations," by Th. Herbert and G. K. Stuckert, 45th Meeting of the American Physical Society, Division of Fluid Dynamics, Tallahassee, Florida (November 1992).

"Stability and Transition on Swept Wings," by G. K. Stuckert, Th. Herbert, and V. Esfahanian, 31st Aerospace Sciences Meeting & Exhibit, Reno, NV, January 11 - 14, 1993.

"Effects of Free-Stream Turbulence on Boundary-Layer Transition," by Th. Herbert, G. K. Stuckert, and V. Esfahanian, 31st Aerospace Sciences Meeting & Exhibit, Reno, NV, January 11 - 14, 1993.

"Parabolized Stability Equations," by Th. Herbert, AGARD Special Course on "Progress in Transition Modelling," Madrid, Spain, March 22-25, 1993.

"Parabolized Stability Equations," by Th. Herbert, AGARD Special Course on "Progress in Transition Modelling," Brussels, Belgium, March 29 - April 1, 1993.

"A PSE Code for the Analysis of Transition in High-Speed Flows," by Th. Herbert, G. K. Stuckert, and N. Lin, Symposium on Hypersonics, Wright-Patterson AFB, Ohio, May 17 - 19, 1993.

"Studies of Boundary-Layer Receptivity with Parabolized Stability Equations," by Th. Herbert and N. Lin, AIAA 24th Fluid Dynamics Conference, Orlando, FL, July 6 - 9, 1993.

"Simulations of Boundary-Layer Transition," by Th. Herbert, Workshop on "End-Stage Transition," Blue Mountain Lake, NY, August 15 - 18, 1993.

"A PSE Code for Transition Analysis," by Th. Herbert, The First Bombardier International Workshop, Montreal, Canada, September 20 - 21, 1993.

"Nonlinear Analysis of Swept Wing Transitional Boundary Layers," by G. K. Stuckert and Th. Herbert, SAE Aerotech '93, Costa Mesa, CA, September 27 - 30, 1993.

"Appearance and Metamorphosis of Klebanoff Modes," by Th. Herbert, 46th Meeting of the APS-DFD, Albuquerque, New Mexico, November 21 - 23, 1993.

"A PSE Code for the Analysis of Receptivity, Stability, and Transition," by Th. Herbert, G. K. Stuckert, and N. Lin, Boeing Commercial Airplane Group, Seattle, Washington, December 13, 1993.

"PSE Analysis of Receptivity and Stability in Swept Wing Flows," by M.

Wang, Th. Herbert, and G. K. Stuckert, 32nd Aerospace Meeting & Exhibit, Reno, Nevada, January 10 - 13, 1994.

"Management of Crossflow Vortices on Swept Wings," by G. K. Stuckert, M. Wang, and Th. Herbert, 12th AIAA Applied Aerodynamics Conference, Colorado Springs, CO, June 20-23, 1994.

"Crossflow-Induced Transition in Compressible Swept-Wing Flows," by M. Wang, Th. Herbert, and G. K. Stuckert, 25th AIAA Fluid Dynamics Conference, Colorado Springs, CO, June 20-23, 1994

"Crossflow-Induced Transition in Swept-Wing Flows," by M. Wang, Th. Herbert, and G. K. Stuckert, 14th IMACS World Congress, Atlanta, GA, July 11-15, 1994.

"PSE Analysis of Transition on Wings and Turbine Blades," by Th. Herbert, Dept. Aeronautics and Astronautics, Purdue University, West Lafayette, IN, November 17, 1994.

"Transition in Open Flow Systems," M. V. Morkovin, E. Reshotko, and Th. Herbert, 47th Meeting of the APS-DFD, Atlanta, GA, November 20 - 22, 1994.

"Transition Mechanisms in Swept-Wing Flows," by M. Wang and Th. Herbert, 47th Meeting of the APS-DFD, Atlanta, GA, November 20 - 22, 1994.

"Linear Wave Motion from Concentrated Harmonic Sources in Blasius Flow," by Th. Herbert and L. M. Mack, 33rd Aerospace Sciences Meeting and Exhibit, January 9-12, Reno, NV.

"Nonparallel Effects in Hypersonic Boundary Layer Stability," by G. K. Stuckert, N. Lin, and Th. Herbert, 33rd Aerospace Sciences Meeting and Exhibit, January 9-12, Reno, NV.

"Boundary Layer Receptivity to Freestream Vortical Disturbances," by N. Lin, G. K. Stuckert, and Th. Herbert, 33rd Aerospace Sciences Meeting and Exhibit, January 9-12, Reno, NV.

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